

## ME 4811

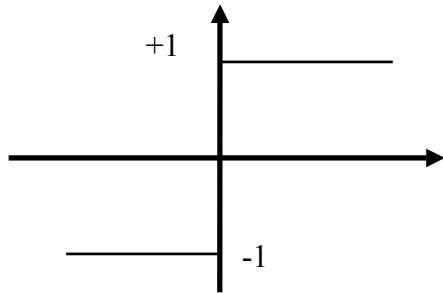
### Lab #6: Time Optimal Control

Consider the second order system

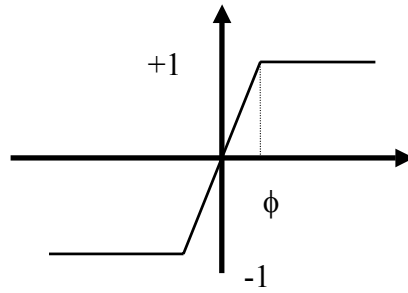
$$\ddot{x} = u \quad \text{where} \quad |u| \leq 1.$$

#### Do the following:

1. Implement, using Simulink, the time optimal control described in the notes. Use two sets of initial conditions so that you get both control actions  $u=\{-1,+1\}$  and  $u=\{+1,-1\}$  developed in class. Plot the response in  $(x_1, x_2)$  coordinates as in Figure 30 in the notes. Does your system follow the switching line at the end? Plot the time history of the control law  $(u, t)$ , does it behave as it should?
2. If there is “chattering” in the control effort  $u$ , eliminate it by appropriately smoothing the sign function as shown in the figures. You may want to use other smoothing functions, such as deadband, etc. Use a “boundary layer” thickness that is small enough so that convergence of  $x$  to zero is still achieved in minimal time.



Pure switch



Smooth switch with boundary layer  
thickness equal to  $\phi$

3. Compare your time optimal control with a regular pole placement control. Vary the poles and compare the various responses. Is your time optimal control essentially a high gain control system?
4. Assume that only  $x$  is measurable and build a full order observer. Simulate your time optimal control with the observer combination. Where should you place the observer poles? Depending on the observer poles, do you still need smaller or larger boundary layer thickness?